

## Status of supersymmetric grand unified theories<sup>\*</sup>

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**Abstract** : We begin with a brief discussion of the building blocks of supersymmetric grand unified theories. We recall some of the compelling theoretical reasons for viewing supersymmetric grand unification as an attractive avenue for physics beyond the standard model. This is followed by a discussion of some of the circumstantial evidence for these ideas.

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### 1. Introduction

The standard model of the strong and electro-weak interactions is based on a Lagrangian field theory of quark, leptonic, scalar Higgs and gauge bosonic degrees of freedom [1,2]. Central to the standard model are the principles of gauge invariance and its spontaneous symmetry breakdown *via* the Higgs mechanism. The standard model predicts the existence of a scalar Higgs particle which is the remnant of the Higgs mechanism by which the gauge bosons of the broken generators of  $SU(2)_L \times U(1)_Y$  become massive when the gauge symmetry is broken down to the residual  $U(1)_{em}$ . The mass of the yet to be discovered Higgs boson is not fixed therein, but is bounded from below from present day experiments and from above by requirements of vacuum stability.

Indeed, if the standard model were to be vindicated by the discovery of the Higgs grand unification appears to be a path to go beyond the energies where the standard model is the correct theory, while continuing to be based on these principles. There would then be a unification scale  $M_G \sim 10^{16}$  GeV suggested by gauge coupling unification, above which

<sup>\*</sup> To the Memory of Prof. Abdus Salam.

physics would be described by a grand unified theory [3] based on a gauge group  $G$ . Such a theory would then make a whole host of predictions and simplifications of our understanding of fundamental phenomena. A compelling goal of theoretical physics is to replace what are the engineering aspects of the standard model by a fundamental theory; for example arbitrary parameters of the standard model, hitherto fixed by experiment, would then be explained as consequences of a unified and symmetric structure. Furthermore, within grand unified theories, one uncovers highly desirable properties such as anomaly free representations of certain grand unified groups. One expects the unification of hitherto unrelated quantum numbers such as baryon and lepton numbers. These in turn imply concrete low-energy predictions which can be confronted by experimental and/or observational information.

The presence of disparate scales in the theory,  $M_G$  and the weak scale  $M_W \sim 174$  GeV, expected to be separated by more than ten orders of magnitude, would render the mass of the Higgs scalar of the electro-weak model  $\sim M_W$ , unnatural-natural. Should the Higgs scalar be elementary, then one manner in which it would remain naturally at the weak scale is due to cancellation of divergences as in supersymmetric unified models [4,5]. Supersymmetry [6] is the only symmetry that has non-trivial commutation relations with the generators of the Lorentz group and is a fermionic object that interchanges bosonic and fermionic degrees of freedom. Although supersymmetry does not appear to be manifest, it could be broken softly while preserving all the desirable properties of supersymmetric theories. Models with softly broken susy are popular and significant experimental effort will be made to test the predictions of these models.

The final frontier that still remains to be explored is a framework within which a consistent incorporation of the gravitational interactions is successful. Whereas it has not been possible to replace the Einstein theory by a quantum version due to bad ultra-violet behaviour, supergravity possesses improved ultra-violet properties [4]. String theories [7] often contain supergravity in their low energy spectrum and as a result supersymmetric unification is a favored candidate for these reasons as well.

Other significant avenues exist for the exploration of these theories. Note, for instance, non-perturbative aspects of the theory such as the possibility of finding topological defects at the time of spontaneous symmetry breakdown when combined with standard big bang cosmology imply specific constraints on grand unified models. Examples of such defects are monopoles, cosmic strings and domain walls.

The task of this talk is to briefly summarize the building blocks of supersymmetric grand unification and recall the main circumstantial evidence for the program. The most significant advance from the experimental direction has come with the precision measurements of the gauge coupling constants at the  $Z^0$  factory LEP [8] (and SLC) and the discovery of the top-quark at the Tevatron [9] by the CDF collaboration and confirmed by the D0 collaboration. These advances place significant constraints on scenarios of unification for a start. More spectacular is the fact that certain scenarios of unification

predicted that the top-quark mass would have to be sufficiently large and roughly in the range where it has been found. Note that combinations of theoretical tools such as the requirement of infra-red fixed point structure of Yukawa couplings as well as finiteness also accommodate top-quark masses in this range. Challenges today lie in spotting the first traces of the supersymmetric partners of the known particles, *e.g.* figuring out search strategies for these for future collider experiments as well as at non-accelerator experiments.

## 2. Spontaneously broken gauge theories

Whereas the gauge invariance of the standard model rests on the gauge group  $SU(3)_C \times SU(2)_L \times U(1)_Y$ , with the quark, lepton [matter] fields and Higgs fields transforming in a specific manner under the gauge group, at low energies, the  $SU(2)_L \times U(1)_Y$  is spontaneously broken to the  $U(1)_{em}$  subgroup at the weak scale *via* the Higgs mechanism. The result is that three of the gauge bosons,  $W^\pm$  and  $Z^0$  pick up masses at the weak scale as does the neutral Higgs scalar. The fermions become massive through the Yukawa couplings to the scalars since the vacuum expectation value  $\langle \phi \rangle \neq 0$ .

It is possible to envisage a scenario wherein this is embedded in a larger group  $G$ , which would be the basis of the gauge invariance of a theory manifest above a unification scale  $M_G$ , below which it would be spontaneously broken *via* the Higgs and possibly some other mechanism to a sub-group large enough to contain the standard model (in a multi-step scenario), which would then be further broken down to the standard model gauge group at various stages.

Circumstantial evidence for this, is found from the renormalization group evolution of the gauge coupling constants of the standard model gauge group, which appears to bring them all together at a large scale  $M_G \sim 10^{16}$  GeV when *the normalization on the hypercharge coupling constant as required by grand unification is imposed*.

Indeed, the arrival at the structure of fundamental interactions from renormalization group flow has a predecessor in the example of asymptotic freedom in deep inelastic scattering experiments and thus gauge coupling unification is an extremely encouraging sign that grand unified theories are the right step for a theory of fundamental interactions. Earliest examples of grand unification were provided by those based on the groups  $SU(4) \times SU(2) \times SU(2)$ ,  $SU(5)$  and  $SO(10)$ .

Grand unification, of course, implies more than just the coming together of the gauge coupling constants. One would be gratified if it were possible to unify the particle content of the theory as well. Indeed, simplifying features of grand unification include embedding several of the matter fields of the standard model into irreducible representations of the underlying gauge group. That such an embedding should at all be possible is an astonishing property of grand unified theory : furthermore, it has the capacity to explain the charge ratios for the elementary fermions in terms of simple group theory. It turns out that  $SO(10)$  [10], for instance, still remains one of the most elegant unification

groups, with an entire standard model family and a right handed neutrino accommodated in a single 16 dimensional representation.

Whereas grand unified theories are based on local Lagrangian field theories possessing symmetries, it is then important to address the question of anomalies in such theories. Compelling theoretical reasons for viewing grand unification as a consistent road to physics beyond the standard model include the fact that several grand unified groups ensure the vanishing of anomalies of gauge currents from the very nature of their representations; *e.g.*, for any irreducible representation of  $SO(10)$ ,  $Tr(Y)$  and  $Tr(Y^3)$  vanish automatically [3], where  $Y$  is the hypercharge generator. Thus anomaly cancellation which may appear somewhat mysterious in the standard model is natural in grand unification; it may be worth noting that while the structure of the strong interaction was arrived at through the analysis of the anomaly in  $\pi^0 \rightarrow 2\gamma$  [2], the structure of theories beyond the standard model may also be uncovered from such considerations of anomalies. In addition, global anomalies are related to the centre of the gauge group :  $Z_2$  in the case of  $SU(2)$ ,  $SO(10)$  and  $Z_3$  in  $E_6$ .

In turn, processes involving transitions from one set of matter fields to another predicting, say the decay of the proton at measurable rates are intrinsic features of unification. The continued failure of the proton to decay within present day experiments in turn implies constraints on scenarios of grand unification [11].

Note that whereas in the standard model, the field content forbids a Dirac mass for the neutrinos since the right handed neutrino is absent and Majorana mass is forbidden by the conservation of lepton number. In grand unified models, neither of these principles is respected and a wide variety of possibilities exists for the generation of neutrino masses. However, far from being arbitrary, it should be possible to uncover information regarding the structure of unified theories from accurate determination of small and eventually large neutrino masses and mixing angles, *viz.*, neutrino masses may be viewed as bearing an imprint on the structure of grand unification and the nature of the breakdown of unification [12].

### 3. Supersymmetric unification

Supersymmetry is the unique symmetry that has non-trivial commutation relations with the generators of the Lorentz group. Supersymmetries enjoy non-trivial anti-commutation relations amongst each other. Their action on representations of the supersymmetry algebra interchange the statistics between the members. Linear representations of the supersymmetry algebra in relativistic field theory are realized in the Wess-Zumino model [6]. Important representations include chiral multiplets and vector multiplets, which form the basis of the extension of the standard model to various supersymmetric versions of the standard model. Since supersymmetry is not manifest in nature, it must be broken, either spontaneously or explicitly. It appears that the second option is more favored, certainly more popular, wherein supersymmetry is broken explicitly but softly. The requirement of

soft supersymmetry breaking is in accordance with the requirement of the well-known properties of supersymmetric models including the cancellation of quadratic mass divergences for scalars.

In the context of grand unified model building, the existence of scales  $M_W$  and  $M_G$  separated by several orders of magnitude renders the mass of the elementary Higgs of the standard model unstable and would drive it to the unification scale, without an un-natural fine tuning of parameters of the Lagrangian. The cancellation of quadratic divergences in manifestly and softly-broken supersymmetric theories renders supersymmetric versions of grand unified models attractive candidates for unification. The program of writing down a supersymmetric version of the standard model, which is then embedded in a grand unified scheme, [alternatively, a supersymmetric version of a grand unified scheme] may be realized by replacing every matter and Higgs field, by a chiral superfield whose members carry the same gauge quantum numbers, and by replacing every gauge field, by a vector super-multiplet. Supersymmetry also requires that the standard model Higgs doublet is replaced by two Higgs multiplets. This in turn leads to the introduction of another parameter  $\tan \beta$  which is defined as the ratio of the vacuum expectation values of these two Higgs fields,  $v_2/v_1$  where  $v_2$  and  $v_1$  are the vacuum expectation value of the Higgs fields that provide the mass for the up-type quark and the down-type and charged leptons respectively. All the interactions of the resulting model may then be written down once the superpotential is specified. Note that gauge invariance and supersymmetry allow the existence of a large number of couplings in the effective theory that would lead to proton decay at unacceptably large rates. An *ad hoc* symmetry called *R*-parity is imposed on the resulting model which eliminates these undesirable couplings and such a version has received the greatest attention for supersymmetry search. More recently models have been and are being considered where *R*-parity is partially broken in order to study the implications to collider searches. However such models are constrained by bounds on flavor changing neutral currents as well as by the standard CKM picture, also as it applies to CP violating phases.

In what follows we recall some of the essential successes of the recent investigations [13] in the theory of supersymmetric unification. This was spurred by the confrontation of the ideas of unification by the precision measurements of the gauge couplings of the standard model at the LEP [14]. A highly simplified understanding of this feature may be obtained from a glance at the non-loop evolution equation for the standard model gauge couplings, more correctly the gauge couplings of the minimal supersymmetric standard model assuming that the effective supersymmetry scale is that of the weak scale, with

$$t = \ln \mu: \quad d\alpha_i = \alpha_i^2 b_i, \quad b_1 = 33/5, \quad b_2 = 1, \quad b_3 = -3, \quad \text{where we have assumed three generations. One may then integrate these equations to obtain: } \frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_i(M_G)} + \frac{1}{2\pi} \ln \frac{M_G}{M_Z}. \quad \text{One may then use the accurately known value of } \alpha_{em}(M_Z) = 1/128, \text{ with}$$

the identity  $1/\alpha_{em} = 5/3\alpha_1 + 1/\alpha_2$  which accounts for the normalization imposed by unification; and the values of  $\alpha_3(M_Z) \approx 0.12$  to solve for the unification scale  $M_G$  and the unified coupling constant  $\alpha_G = \alpha_{1,2,3}(M_G)$ . One then has a prediction for  $\sin^2 \theta_w$  at the weak scale which comes out in the experimentally measured range. Sophisticated analysis around this highly simplified picture up to two and even three loops taking into account the Yukawa couplings of the heaviest generation which contribute non-trivially at the higher orders, threshold effects, *etc.*, vindicate this picture of gauge coupling unification which today provides one of the strongest pieces of circumstantial evidence for grand unification [15].

Predictions arising from (supersymmetric) unification such as for the mass of the top-quark have been vindicated experimentally. It turns out that unification based on  $SO(10)$  is a scheme with great predictive power not merely in the context of top-quark mass but also with implications for the rest of the superparticle spectrum. The primary requirement that is imposed is that the heaviest generation receives its mass from a unique coupling in the superpotential  $h \mathbf{16.16.10}$  where the  $\mathbf{16}$  contains a complete generation and the complex  $\mathbf{10}$  the two electroweak doublets [16]. When the Yukawa couplings of the top and  $b$ -quarks and the  $\tau$ -lepton are evolved down to the low energy and  $\tan \beta$  pinned down from the accurately known  $\tau$ -mass, one has a unique prediction for the  $b$  and top-quark masses for a given value of  $h$ . If  $h$  is chosen so as to yield  $m_b(m_b)$  in its experimental range, the top-quark mass is uniquely determined up to these uncertainties. Now  $\tan \beta \approx m_t/m_b$ , and the top- $b$  hierarchy is elegantly explained in terms of this ratio coming out large naturally.

It is truly intriguing that this picture yields a top-quark mass in its experimental range, with  $\alpha_s$  in the range of the LEP measurements despite the complex interplay between the evolution equations involved, the determination of the unification scale, running of QCD couplings below the weak scale. Note that this requires that the top-Yukawa coupling must also come out of order unity at  $M_Z$ . It is also worth noting that due to the nature of the evolution equations and competition between the contributions to these from the gauge and Yukawa couplings, this number  $m_t(m_t)$  lies near a quasi-fixed point of its evolution, *viz.* there is some insensitivity to the initial choice of  $h$  [17]. Moreover, if the  $SO(10)$  unification condition is relaxed to an  $SU(5)$  one where only the  $b$ -quark and  $\tau$ -lepton Yukawa couplings are required to unify at  $M_G$ ,  $m_t(m_t)$  comes out in the experimental range while preserving  $m_b(m_b)$  in its experimental range for  $\tan \beta$  near unity. In this event also the top-quark Yukawa coupling lies near a quasi-fixed point which is numerically larger compensating for the smaller value of  $\sin \beta$  that enters the expression for its mass :  $m_t = h / \sin \beta$  174 GeV. Another interesting connection arises in this context between the values of the Yukawa couplings at unification and that of the gauge coupling when one-loop finiteness and reduction of couplings is required : such a program also yields top-quark masses in the experimental range [18].

Besides the vindication of top-quark discovery predicted by susy guts, another strong test takes shape in the form of its prediction for the scalar spectrum. In the MSSM

the mass of the lightest scalar is bounded at tree level by  $M_Z$  since all quartic couplings arise from the  $D$ -term in the scalar potential. The presence of the heavy top-quark enhances the tree-level mass, but the upper bound in these models is no larger than 140 GeV.

Other predictions for softly-broken susy models arise when a detailed analysis of the evolution equations of all the parameters of the model are performed and the ground state carefully analyzed. In the predictive scheme with  $SO(10)$  unification, the model is further specified by  $M_{1/2}$ ,  $m_0$  and  $A$ , the common gaugino, scalar and tri-linear soft parameters [5]. It turns out that in this scheme  $M_{1/2}$  is required to come out to be fairly large, at least  $\sim 500$  GeV implying a lower bound on the gluino mass of a little more than a TeV and providing a natural explanation for the continuing absence of observation of susy particles from scenarios based on radiative electro-weak symmetry breaking [19]. [An extensive study of the NMSSM with  $SO(10)$  conditions has also been performed [20]]. Considerably greater freedom exists when the  $SO(10)$  boundary condition is relaxed [21]. In summary many predictions and consistency of the MSSM and its embedding in a unified framework have been vindicated; however, it is important to continue theoretical investigations and checks to the consistency of these approaches and extensions to include the lighter generations [22].

#### 4. Monopoles

This discussion is somewhat off the main stream of the discussion above. Furthermore, if one were to discuss spontaneously broken gauge field theory at finite temperatures, when temperatures reach the scale of symmetry breaking, then phase transitions are expected to occur which restore broken symmetries. Indeed, at such phase transitions, one expects the formation of topological defects which may be characterized by certain topological properties known as homotopy groups of the coset space :  $G/H$ , where  $G$  is the gauge group that is broken to the subgroup  $H$ . Examples of topological defects are domain walls, strings and monopoles, which may have been produced in the early universe as the universe cooled to present temperatures. This is an example of an aspect of gauge field theory that is outside the realm of perturbation theory. However, certain interesting preliminary investigations indicate that monopoles are inconsistent as asymptotic states; they are confined even if in the topologically parallelizable sector the gauge theory serving as non-abelian basis to the classically acceptable monopole solutions is broken [23] [Other examples of standard model physics that the outside this realm is that of the formation of fermion condensates that are required to spontaneously break chiral symmetry that lead to the generation of massless pions when the quark masses are set to zero].

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